

MATHEMATICS-I



Deepak Singh



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FOREWORD

Engineering has played a very significant role in the progress and expansion of mankind and society for centuries. Engineering ideas that originated in the Indian subcontinent have had a thoughtful impact on the world.

All India Council for Technical Education (AICTE) had always been at the forefront of assisting Technical students in every possible manner since its inception in 1987. The goal of AICTE has been to promote quality Technical Education and thereby take the industry to a greater heights and ultimately turn our dear motherland India into a Modern Developed Nation. It will not be inept to mention here that Engineers are the backbone of the modern society - better the engineers, better the industry, and better the industry, better the country.

NEP 2020 envisages education in regional languages to all, thereby ensuring that each and every student becomes capable and competent enough and is in a position to contribute towards the national growth and development.

One of the spheres where AICTE had been relentlessly working from last few years was to provide high-quality moderately priced books of International standard prepared in various regional languages to all it's Engineering students. These books are not only prepared keeping in mind it's easy language, real life examples, rich contents and but also the industry needs in this everyday changing world. These books are as per AICTE Model Curriculum of Engineering & Technology – 2018.

Eminent Professors from all over India with great knowledge and experience have written these books for the benefit of academic fraternity. AICTE is confident that these books with their rich contents will help technical students master the subjects with greater ease and quality.

AICTE appreciates the hard work of the original authors, coordinators and the translators for their endeavour in making these Engineering subjects more lucid.

(Anil D. Sahasrabudhe)

Acknowledgement

The author is grateful to AICTE for their meticulous planning and execution to publish the technical book for Diploma students.

I sincerely acknowledge the valuable contributions of the reviewer of the book Dr. Pradip Nandlal Joshi, for making it students' friendly and giving a better shape in an artistic manner.

This book is an outcome of various suggestions of AICTE members, experts and authors who shared their opinion and thoughts to further develop the engineering education in our country.

It is also with great honour that I state that this book is aligned to the AICTE Model Curriculum and in line with the guidelines of National Education Policy (NEP) -2020. Towards promoting education in regional languages, this book is being translated in scheduled Indian regional languages.

Acknowledgements are due to the contributors and different workers in this field whose published books, review articles, papers, photographs, footnotes, references and other valuable information enriched us at the time of writing the book.

Finally, I like to express my sincere thanks to the publishing house, M/s. Khanna Book Publishing Company Private Limited, New Delhi, whose entire team was always ready to cooperate on all the aspects of publishing to make it a wonderful experience.

Deepak Singh

Preface

The book titled “Mathematics-I” is an outcome of teaching Mathematics for Engineering students. The initiation of writing this book is to expose basic concepts of Mathematics for Diploma engineering students to the fundamentals of Mathematics as well as enable them to get an insight of the subject. Keeping in mind the purpose of wide coverage and also to provide essential supplementary information, Author included the topics recommended by AICTE, in a very systematic and logical manner throughout the book. Efforts have been made to explain the fundamental concepts of the subject in the simplest possible way.

During the process of preparation of the manuscript, I have considered the various standard text books and accordingly sections like create inquisitiveness, solved and supplementary problems etc., have developed. While preparing the different sections emphasis has also been laid on comprehensive synopsis of formulae for a quick revision of the basic principles. Apart from Tapping into areas of student interest, the author provides an ample supply of examples and rich exercises. Found in every Unit, realistic applications draw students into the discipline to help them to generalize the material and apply it to new and novel situations. To further spark student interest, meticulously drawn graphs and illustrations appear throughout the text.

In addition, besides some essential information for the users under the heading “Know More” the Author have clarified some essential basic information for further readings.

As far as the present book is concerned, “Mathematics-I: is meant to provide a thorough grounding in the subject on the topics covered. This part of the Mathematics-I will prepare students to apply the knowledge of Trigonometry, Calculus and Algebra to address the related aroused questions. The subject matters are presented in a constructivemanner.

The Author sincerely hope that the book will inspire the students to learn and discuss the ideas behind basic principles of Trigonometry, Calculus and Algebra to the develop solid basis of the subject. I would be thankful to all beneficial comments and suggestions which will contribute to the improvement of the future editions of the book. It gives me immense pleasure to place this book in the hands of the teachers and students. It was indeed a big pleasure to work on different aspects covering in the book.

Deepak Singh

Outcome Based Education

For the implementation of an outcome-based education the first requirement is to develop an outcome-based curriculum and incorporate an outcome-based assessment in the education system. By going through outcome-based assessments evaluators will be able to evaluate whether the students have achieved the outlined standard, specific and measurable outcomes. With the proper incorporation of tools of outcome-based education there will be a certain commitment to achieve a minimum standard for all learners without giving up at any level. At the end of the program running with the aid of outcome-based education, a student will be able to arrive at the following outcomes:

- PO-1: Basic and Discipline specific knowledge: Apply knowledge of basic mathematics, science and engineering fundamentals and engineering specialization to solve the engineering problems.
- PO-2: Problem analysis: Identify and analyze well-defined engineering problems using codified standard methods.
- PO-3: Design/ development of solutions: Design solutions for well-defined technical problems and assist with the design of systems components or processes to meet specified needs.
- PO-4: Engineering Tools, Experimentation and Testing: Apply modern engineering tools and appropriate technique to conduct standard tests and measurements.
- PO-5: Engineering practices for society, sustainability and environment: Apply appropriate technology in context of society, sustainability, environment and ethical practices.
- PO-6: Project Management: Use engineering management principles individually, as a team member or a leader to manage projects and effectively communicate about well-defined engineering activities.
- PO-7: Life-long learning: Ability to analyze individual needs and engage in updating in the context of technological changes.

Course Outcomes

After completion of the course the students will be able to:

- CO-1: Apply trigonometry and related basic concepts to solve applied technical problems.
- CO-2: Demonstrate the ability to algebraically analyse basic functions used in Trigonometry.
- CO-3: Use basic concepts of Differential Calculus to solve engineering related problems.
- CO-4: Interpret the derivative of a function graphically, numerically and analytically.
- CO-5: Demonstrate the ability to model real -life scenarios using functions.
- CO-6: Communicate mathematical thinking coherently and clearly to students, peers, and others.
- CO-7: Solve engineering related problems based on concepts of Algebra.

Course Outcome	Expected Mapping with Program Outcomes (1- Weak Correlation; 2- Medium correlation; 3- Strong Correlation)						
	PO-1	PO-2	PO-3	PO-4	PO-5	PO-6	PO-7
CO-1	3	2	1	-	-	-	1
CO-2	3	2	-	-	-	-	1
CO-3	3	2	1	-	-	-	1
CO-4	2	2	-	-	-	-	1
CO-5	2	2	1	-	-	-	-
CO-6	2	2	1	-	-	-	2
CO-7	2	2	-	-	-	-	-

Abbreviations and Symbols

List of Abbreviations

Abbreviations	Full form
CO	Course Outcomes
cm	Centimeter
PO	Program Outcomes
t-Ratio	Trigonometrical ratio
UO	Unit Outcomes
UV	Ultra Violet
All STC	
All	All trigonometric t-ratios are positive
S	Sine and Cosec t-ratios are positive
T	Tan and Cot t-ratios are positive
C	Cos and Sec t-ratios are positive

List of Symbols

Symbols	Description
1^R	1 Radian
1^G	1 Grade
	1 Degree
1^0	1 Degree
$1'$	1 Minute
$1''$	1 Second
π	Pai
D_f	Domain of a function f
R_f	Range of a function f
$x \rightarrow a$	x approaches to a
\cup	Union
\cap	Intersection

Symbols	Description
\subset	Subset
\in	Belongs to
$[a, b]$	Closed interval
(a, b)	Open interval
$[a, b)$	Close – open interval
$(a, b]$	Open – close interval
ϕ	Empty Set
$[x]$	Greatest Integer
$\frac{d}{dx}(y) = \frac{dy}{dx}$	Derivative of w.r.t. x
i	Iota
\bar{z}	Conjugate of Z
$ z $	Modulus of Z
$\text{Re}(z)$	Real part of Z
$\text{Im}(z)$	Imaginary part of Z
$\text{arg}(z)$	Argument of Z
$\text{amp}(z)$	Amplitude of Z
$\text{cis}\theta$	$\cos\theta + i\sin\theta$
$n!$	n factorial OR factorial n
${}^n P_r$	Number of permutations of n different objects taken r at a time
${}^n C_r$	Number of Combination of n different objects taken r at a time

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Guidelines for Teachers

To implement Outcome Based Education (OBE) knowledge level and skill set of the students should be enhanced. Teachers should take a major responsibility for the proper implementation of OBE. Teachers are suggested to use the special instructional strategies to accelerate the attainment of the various outcomes in this course. Some of the responsibilities (not limited to) for the teachers in OBE system may be as follows:

- Within reasonable constraint, they should manipulate time to the best advantage of all students.
- Massive open online courses (MOOCs) may be used to teach various topics/subtopics.
- Different types of teaching methods and media may be employed to develop the outcomes.
- About 10-15% of the topics/sub-topics which is relatively simpler or descriptive in nature is to be given to the students for self-directed learning and assess the development of the UOs/COs through classroom presentations.
- Assess the students only upon certain defined criterion without considering any other potential ineligibility to discriminate them.
- Guide student(s) in undertaking micro-projects.
- Ensure to grow the learning abilities of the students to a certain level before they leave the institute.
- Employ ICT Based Teaching Learning (Video Demonstration, Blog, Face book, Mobile learning)
- Teachers need to ensure to create opportunities and provisions for co-curricular activities.
- Encourage the students to develop their ultimate performance capabilities.
- Facilitate and encourage group work and team work to consolidate newer approach.
- Follow Blooms taxonomy in every part of the assessment.

Bloom's Taxonomy

Level	Teacher should Check	Student should be able to	Possible Mode of Assessment
Creating	Students ability to create	Design or Create	Mini project
Evaluating	Students ability to Justify	Argue or Defend	Assignment
Analysing	Students ability to distinguish	Differentiate or Distinguish	Project/Lab Methodology
Applying	Students ability to use information	Operate or Demonstrate	Technical Presentation/ Demonstration
Understanding	Students ability to explain the ideas	Explain or Classify	Presentation/Seminar
Remembering	Students ability to recall (or remember)	Define or Recall	Quiz

Guidelines for Students

Students should take equal responsibility for implementing the OBE. Other than the classroom learning, following are the suggested student-related co-curricular activities which can be undertaken to accelerate the attainment of the various outcomes in this course. Some of the responsibilities (not limited to) for the students in OBE system are as follows:

- Students should be well aware of each UO before the start of a unit in each and every course.
- Students should be well aware of each CO before the start of the course.
- Students should be well aware of each PO before the start of the program.
- Students should think critically and reasonably with proper reflection and action.
- Learning of students should base on real world problems relevant to content of the unit using free tutorials available on the internet.
- Learning of the students should be connected and integrated with practical and real-life consequences.
- Students should be well aware of their competency at every level of OBE.

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